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Research Article

NUCLEON-NUCLEON INTERACTION IN INFINITE NUCLEAR MATTER

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ABSTRACT

The thermodynamic properties of Rhenium ($^{186}_{75}$Re) using Nucleon-Nucleon (*NN*-Interaction) via a scattering potential $V = \beta\chi^3 + \gamma\chi^4$ in addition to the unperturbed Hamiltonian H_o is studied. The second quantization approach was used to calculate the Hamiltonian of the system. The heat capacity variation with temperature showed a second order phase transition at $T_c = 0.144$ K. The variation of heat capacity with excitation energy exhibits a linear relationship but with increasing gradients at higher values of constant temperatures and attains a maximum value corresponding to the critical transition temperature T_c. This shows that the maximum specific heat at all excited states occurs at the critical transition temperature T_c due to thermal excitation. These results ascertain the presence of low temperature property of superconductivity in the Fermi system.

Keywords: *Superfluidity, Phase Transition and Critical Transition Temperature*

INTRODUCTION

The discovery of neutron by Chadwick (1932) has completed more or less the structure studies of the nucleus. After this discovery, the nucleus was proposed to be composed of neutrons and protons, which have almost the same mass. Protons and neutrons commonly referred as nucleons are spin-half particles, which obey Fermi-Dirac statistics (Roy and Nigam, 1967). Previous studies have indicated that the nuclear forces provide the strength of the atomic nuclei while the electron interacts with the nucleus as a bar magnet (Kanarev, 2003). Remarkably, nature built the nucleus in such a way that a neutron is in between the protons to minimize repulsion and enhance stability (Kanarev, 2003). This requirement is not easy to meet hence many large atoms have more neutrons than protons.

The recent development of renewed interest has been towards the calculation of the properties of nuclear matter and large finite nuclei using different types of nucleon-nucleon interactions (Lassey, 1972; Khanna and Barhai, 1975; Moszkowski, 1970; Dean and Hjorth-Jensen, 2003). Particularly, the ground state energy of the nuclear matter has been calculated using various types of nucleon-nucleon potentials. A velocity dependent effective potential of *s*-wave interaction with one free parameter was proposed (Dzhibuti and Mamasa, 1969) to calculate the properties of nuclear matter, especially binding energy and radii of different nuclei from ^{4}He to ^{208}Pb.

A set of nucleon-nucleon interaction is characterized by the existence of a strongly repulsive core at short distances, with a characteristic radius $\simeq 0.5\ f$m (Dean and Hjorth-Jensen, 2003). The interaction obeys several fundamental symmetries, such as translational, rotational, spatial reflection, time-reversal invariance, and exchange symmetry. It has also a strong dependence on quantum numbers such as total spin S and isospin T, through the nuclear tensor force that arises, for instance, from one-pion exchange. It also depends on the angles between the nucleon (pairs) spins and separation vector. The tensor force thus mixes different angular momenta L of the two-body system, that is, it couples two-body states with total angular momentum $J=L-1$ and $J=L+1$. For instance, for a proton-neutron two-body state, the tensor force couples the states ${}^3S_1$and ${}^3D_1$, where the standard spectroscopic notation ${}^{2S+1}L_J$has been used.

Theory

This study has developed a nucleon-nucleon interaction theory based on quantization in calculating the thermodynamic properties from the Hamiltonian of the perturbed system. Two nucleons interact with each other harmonically. We add anharmonicity through H^1 which is the perturbation. Hence the assembly of nucleons develops an anharmonic potential. Thus to calculate the energy of the system, we shall write the perturbed Hamiltonian H as,

$$H = H_0 + V \qquad 1$$

where H_0 is the unperturbed harmonic Hamiltonian, expressed as,

$$H_0 = \frac{p^2}{2m} + \frac{1}{2}kx^2 \qquad 2$$

and which is the perturbation that causes anharmonicity in the harmonic interactions between the nucleons and interaction potential is given as,

$$V = \beta\mathcal{X}^3 + \gamma\mathcal{X}^4 \qquad 3$$

The parameters β and γ are the constants of perturbation.

The methods of second quantization and many-body perturbation theory are used to calculate the total energy E_n of the nucleus. From the concept of many-body systems we can write,

$$\langle n|H|n\rangle = \langle n|H_0|n\rangle + \langle n|H^1|n\rangle \qquad 4$$

The eigenvalues and eigenfunctions of the unperturbed Hamiltonian are expressed as:

$$\langle n|H_0|n\rangle = E_0\langle n|n\rangle \qquad 5$$

where,

$$E_0 = \left(n + \frac{1}{2}\right)\hbar\omega, \quad n=0,1,2,3\ldots \qquad 6$$

The perturbed Hamiltonian becomes

$$\langle n|H^1|n\rangle = \frac{\beta}{\alpha^3\sqrt{8}}\langle n|(a + a^+)^3|n\rangle + \frac{\gamma}{4\alpha^2}\langle n|(a + a^+)^4|n\rangle \qquad 7$$

where the H^1 is the equivalent effect due to the perturbing potential.

Expanding Eq(7) and using the Bogoulibov transformation in which the annihilation and creation operators are; $a = u_R\ell_R + v_R\ell_R^+$ and $a^+ = u_R\ell_R^+ + v_R\ell_R$ respectively with the following properties:

$$u_R = v_R = \frac{1}{\sqrt{2}};\ u_R^2 + v_R^2 = 1 \qquad 8$$

By diagonalizing, the total energy of the system can be written as,

$$E_n = E_n^0 + E_n^{/} \qquad 9$$

or

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega + \frac{\gamma}{16\alpha^2}\left(96n^2 + 86n + 49\right) \qquad 10$$

Where $n = 0,1,2,3\ldots$; $\gamma = \frac{\hbar\omega}{a_0^2}$ and $\alpha^2 = \frac{\mu\omega}{\hbar}$.

And that a_0 is the nuclear radius while μ *is* the neutron – proton reduced mass. The energy corresponding to the excited energy states n(0,1,2,3,..) can be expressed with a thermal factor as,

$$E_n = \left[\left(n + \frac{1}{2}\right)\hbar\omega + \frac{\gamma}{16\alpha^2}\left(96n^2 + 86n + 49\right)\right]e^{-\frac{\hbar\omega}{\kappa T}} \qquad 11$$

Heat Capacity C

The heat capacity can be expressed as

$$C = \left(\frac{\partial E_n}{\partial T} \right) \qquad 12$$

$$C = \hbar\omega \frac{E_0}{\kappa T^2} e^{-\frac{\hbar\omega}{\kappa T}} \qquad 13$$

For n = 0, E_0 = Ground state energy

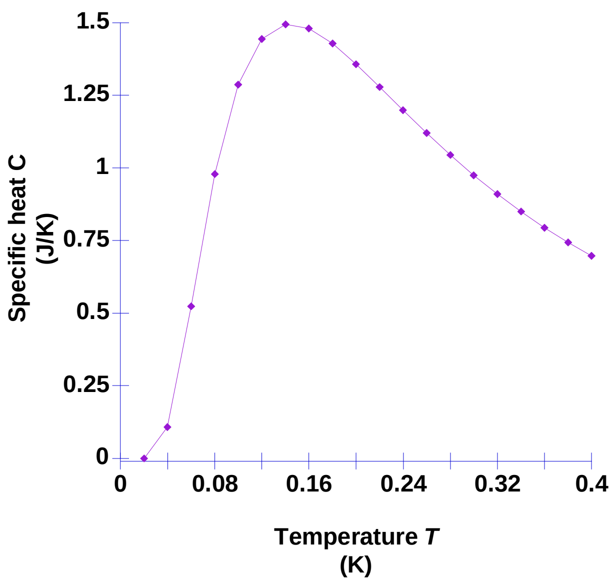
RESULTS AND DISCUSSION

The heat capacity and excitation energy have been calculated using various parameters provided in Table 1 below.

Table 1: Parameters used and their corresponding values

Parameter	Value
Planck's constant	h =6.626x10^{-34} J-sec
Boltzmann constant	k_B =1.3806504x10^{-23}J/K
Nuclear radius	$a_o = 1.3\text{x}10^{-15} A^{1/3} cm$
Vibrational frequency	ω = 6x10 ^{10}s^{-1}
Neutron-Proton reduced mass	μ = 8.369 x 10^{-28} Kg

The graph in Figure 1 shows the variation of heat capacity with temperature changes. The heat capacity for $^{186}_{75}$Re increases non-linearly with increase in temperature, and attains a maximum value corresponding to the critical transition temperature at T_c = 0.144 K with a peak value of 1.5 J/K.



Research Article

Figure 1: Variation of Heat Capacity with Temperature for $^{186}_{75}$Re

These results indicate the possibility of the material to attain superfluidity at a temperature below this critical transition temperature. The heat capacity values are positive mainly due to the presence of Coulomb interaction potential (Moretto *et al.*, 2004) at the vicinity of ground state.

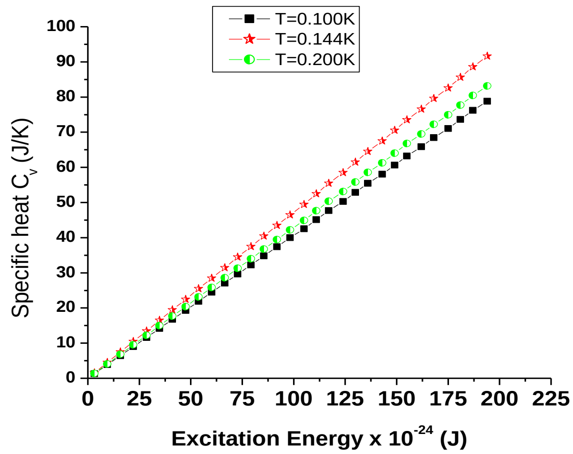


Figure 2: Variation of Heat Capacity with Excitation Energy at constant Temperatures

Using Eq(11), the variation of heat capacity with excitation energy at constant temperature exhibits a linear relationship as shown in Figure 2. The gradients increase sharply with rising excitation energy at higher values of constant temperatures to a maximum gradient corresponding to the critical temperature T_c. This shows that the maximum heat capacity at all excited states occurs at the critical transition temperature T_c and this could be due to thermal excitation effects. These results are evident of Rhenium $^{186}_{75}$Re becoming a superfluid below the critical transition temperature 0.144 K. This transition temperature is reminiscent of the experimental value of Hulm (1954), Hulm and Goodman (1957) of within 0.1 K.

Apart from second quantization approach other theoretical approaches may still be used to ascertain this result.

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